

Electromagnetic annihilation into charged leptons and scattering off nucleons of spin-3/2 Majorana particles

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We compute the cross section for the electromagnetic annihilation into charged leptons, and the electromagnetic scattering off nucleons, of spin-3/2 self-conjugate (Majorana) particles using the general form of the electromagnetic vertex function that was obtained previously for such particles. In addition to the restrictions imposed by common principles such as electromagnetic gauge invariance and hermiticity, the vertex function incorporates the restriction due to the Majorana condition as well as the particular properties related to the spinors in the Rarita-Schwinger representation, and is the counterpart of the so-called *anapole* interaction of spin-1/2 Majorana particles. The formulas obtained for the cross sections share certain similarities with the corresponding results in the spin-1/2 case, but they also reveal some important differences which are pointed out and discussed. The results given here can be useful for applications involving the electromagnetic interactions of spin-3/2 or spin-1/2 Majorana particles in several contexts that have been of interest in the recent literature such as nucleosynthesis and dark matter.

I. INTRODUCTION AND SUMMARY

The study of the electromagnetic processes of electrically neutral particles has been of interest in a variety of physical and astrophysical contexts. This is largely due to the fact that since they do not couple to the photon at the tree-level, their electromagnetic properties can provide a window to higher order effects or sectors of the Standard Model (SM) and its extensions that may be difficult or impossible to probe directly. This is even more so in the particular case of Majorana particles. For example, it has been known for a long time that Majorana neutrinos can have neither electric nor magnetic dipole moments; it can only have the so-called axial charge radius. Results of this type can also be deduced, for example, for the transition moments between two different Majorana neutrinos[1–5], and similar results hold for spin-1 particles[6] and for Majorana particles of arbitrary spin[7]. By the same token, the experimental observation of departures from these results would have implications for some important principles such as Lorentz and gauge invariance, *CPT* and crossing symmetry.

Motivated by this, we recently considered in a general way the electromagnetic properties of spin-3/2 Majorana particles[8], to provide a systematic study that while being helpful for considering questions of fundamental and intrinsic interest, is also useful for phenomenological applications in several areas of current interest. For example, there has been considerable activity recently in the study of the possible effects of gravitinos in several cosmological contexts such as nucleosynthesis and inflation[9–12], and the implications of spin-3/2 particles as dark matter candidates and their detection[13–15]. Many of these effects have to do with the electromagnetic properties of gravitinos, or more generally with the electromagnetic properties of spin-3/2 Majorana particles.

In the present work we calculate the cross section for the electromagnetic annihilation into charged leptons, and the electromagnetic scattering off nucleons, by a spin-3/2 self-conjugate (Majorana) particle using the results obtained in Ref. [8] for the electromagnetic vertex function of such a particle. The analogous results for spin-1/2 Majorana particles, which involve the so-called *anapole* interaction, have been used recently in detailed analysis in Refs. [16, 17] to constrain their properties. As we show, the cross sections in the spin-3/2 case share certain similarities with the corresponding results in the spin-1/2 case, but they also reveal some important differences. The results presented here can be useful in the contexts already mentioned above, and they can be also used to extend to the spin-3/2 case

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the work carried out in Refs. [16, 17] for spin-1/2.

To be more precise, let us summarize the results obtained in Ref. [8] that are most relevant for the present work. Borrowing from the notation introduced there, we denote by $j_\mu(Q, q)$ the matrix element of the electromagnetic current operator $J_\mu^{(\text{em})}(0)$ between two spin-3/2 particle states, with momentum k, k' and spin projection σ, σ' ,

$$j_\mu(Q, q) \equiv \langle k', \sigma' | J_\mu^{(\text{em})}(0) | k, \sigma \rangle, \quad (1.1)$$

where

$$\begin{aligned} q &= k - k', \\ Q &= k + k'. \end{aligned} \quad (1.2)$$

The electromagnetic off-shell vertex function $\Gamma_{\alpha\beta\mu}(Q, q)$ is then defined such that

$$j_\mu(Q, q) = \bar{U}^\alpha(k', \sigma') \Gamma_{\alpha\beta\mu}(Q, q) U^\beta(k, \sigma), \quad (1.3)$$

where $U^\alpha(k, \sigma)$ is a Rarita-Schwinger (RS) spinor.

Then, for the particular case in which the initial and final particle is the same and it is self-conjugate (the *diagonal Majorana case*), which denote by λ , the most general form of the vertex function consistent with Lorentz and electromagnetic gauge invariance, involves at most two independent terms, which can be written in the form

$$\begin{aligned} \Gamma_{\alpha\beta\mu} = & f g_{\alpha\beta} (q^2 \gamma_\mu - q_\mu \not{q}) \gamma_5 + \frac{if}{m_\lambda} (q_\beta \epsilon_{\alpha\mu\nu\lambda} - q_\alpha \epsilon_{\beta\mu\nu\lambda}) q^\nu Q^\lambda \\ & + ig [q^2 (g_{\mu\alpha} q_\beta + g_{\mu\beta} q_\alpha) - 2q_\mu q_\alpha q_\beta], \end{aligned} \quad (1.4)$$

where m_λ is the λ particle's mass and the two corresponding form factors, denoted here by f and g , are real. The term proportional to $\gamma_\mu \gamma_5$ is reminiscent of the axial charge radius term for Majorana neutrinos[1–3], while the other two terms resemble the result that was obtained in Ref. [6] for the electromagnetic vertex function of self-conjugate spin-1 particles [Eq. (4.14) in that reference]. Further conditions exist if some discrete symmetries hold. For example, if CP holds, then $g = 0$, so that only the first two terms in Eq. (1.4), involving the f form factor, can be present. An expression similar to Eq. (1.4) was obtained in Ref. [7], where it was found that the vertex function in this same case (spin-3/2 diagonal Majorana case) consists of at most three terms. However, the distinctive feature of the result obtained in Ref. [8] that we are emphasizing here, is the fact that the coefficients of the first two terms in Eq. (1.4) are not independent, so that at most two form factors are involved. In particular, if the $\gamma_\mu \gamma_5$ is present, then it must be accompanied by the second term in Eq. (1.4). This result is due to the requirement that the vertex function does not mix the genuine spin-3/2 degrees of freedom with the spurious spin-1/2 components of the RS representation. Stated more precisely, if we denote by $(S_{\mu\nu})_{\alpha\beta}$ and $\frac{1}{2}\sigma_{\mu\nu}g_{\alpha\beta}$ the spin-1 and spin-1/2 Lorentz group generators in the RS representation, the above result follows by requiring that these generators do not appear in the vertex function separately, but in the combination $(\Sigma_{\mu\nu})_{\alpha\beta} = (S_{\mu\nu})_{\alpha\beta} + \frac{1}{2}\sigma_{\mu\nu}g_{\alpha\beta}$, and its products.

In this work we use the expression given in Eq. (1.4) as the starting point to compute the cross section for the annihilation process

$$\lambda + \lambda \rightarrow \ell + \bar{\ell}, \quad (1.5)$$

where ℓ stands for a charged lepton (e.g., the electron), and for the scattering process

$$\lambda + f \rightarrow \lambda + f \quad (1.6)$$

where $f = \ell, n, p$ is a charged lepton or a nucleon. We restrict ourselves to the case $g = 0$, that is we assume that in these processes the CP symmetry holds. We begin in Section II with a summary of the notation and conventions, and in particular with our conventions regarding the spin-3/2 particle spinor wave functions. In Section III we carry out the calculation of the annihilation cross section and present the formula for the differential cross section and in Section IV we present the corresponding calculations and results for the scattering process. Because the calculational algebra that involves using the RS spinors can be cumbersome and lengthy, some details of the calculations in those two sections are provided in three appendices. In Section V we discuss the salient features of the results we have obtained and compare them with the cross sections for the analogous processes involving a spin-1/2 Majorana or a spin-1 self-conjugate particle, denoted by χ and V respectively. We close that section with some concluding remarks and outlook regarding the processes $\lambda + \lambda \rightarrow \gamma + \gamma$ and $\lambda + \gamma \rightarrow \lambda + \gamma$. In contrast with the analogous χ processes, the tree-level amplitudes for both of these processes do not vanish and therefore it could be of interest to consider them along similar lines.

II. NOTATION AND PRELIMINARIES

A. Kinematics

1. Scattering process

To establish the notation and conventions let us consider the scattering process

$$\lambda(k) + f(p) \rightarrow \lambda(k') + f(p'), \quad (2.1)$$

where the momentum vectors have components

$$\begin{aligned} k^\mu &= (\omega, \vec{k}), \\ p^\mu &= (E, \vec{p}), \end{aligned} \quad (2.2)$$

with similar notation for the primed counterparts, satisfying the on-shell conditions

$$\begin{aligned} k^2 &= k'^2 = m_\lambda^2, \\ p^2 &= p'^2 = m_f^2. \end{aligned} \quad (2.3)$$

Denoting by q the virtual photon momentum,

$$q = k - k' = p' - p, \quad (2.4)$$

and defining

$$\begin{aligned} Q &= k + k', \\ P &= p + p', \end{aligned} \quad (2.5)$$

the following relations hold,

$$\begin{aligned} k \cdot k' &= m_\lambda^2 - \frac{1}{2}t, \\ Q^2 &= 4m_\lambda^2 - t, \\ p \cdot p' &= m_f^2 - \frac{1}{2}t, \\ P^2 &= 4m_f^2 - t, \end{aligned} \quad (2.6)$$

where

$$t = q^2. \quad (2.7)$$

2. Annihilation process

In the annihilation channel,

$$\lambda(k) + \lambda(k') \rightarrow \ell(p) + \bar{\ell}(p'). \quad (2.8)$$

the virtual photon momentum is

$$q = k + k' = p + p', \quad (2.9)$$

and

$$Q = k - k'. \quad (2.10)$$

Assuming again the on-shell conditions, the following relations then hold

$$k \cdot k' = \frac{1}{2}s - m_\lambda^2, \quad (2.11)$$

$$Q^2 = 4m_\lambda^2 - s, \quad (2.12)$$

where

$$s = q^2. \quad (2.13)$$

B. Spinor conventions and relations

The spin-1/2 Dirac spinors are denoted by u, v , while U, V denote the spin-3/2 Rarita-Schwinger spinors. The Dirac spinors are normalized in the usual way so that in particular the polarization sums are given by the standard formulas

$$\begin{aligned}\rho(p) &\equiv \sum_s u(p, s) \bar{u}(p, s) = \not{p} + m_f, \\ \bar{\rho}(p) &\equiv \sum_s v(p, s) \bar{v}(p, s) = \not{p} - m_f.\end{aligned}\tag{2.14}$$

The spin-3/2 spinors are normalized such that

$$-\bar{V}^\mu V_\mu = \bar{U}^\mu U_\mu = -2m_\lambda,\tag{2.15}$$

where the minus sign in the right-hand-side is analogous to the minus sign in the normalization of the spin-1 polarization vectors, $\epsilon^\mu \epsilon_\mu = -1$. The formulas for the polarization sums corresponding to Eq. (2.14) are

$$\begin{aligned}\rho_{\mu\nu}(k) &\equiv \sum_\sigma U_\mu(k, \sigma) \bar{U}_\nu(k, \sigma) = -(\not{k} + m_\lambda) R_{\mu\nu}(k), \\ \bar{\rho}_{\mu\nu}(k) &\equiv \sum_\sigma V_\mu(k, \sigma) \bar{V}_\nu(k, \sigma) = -(\not{k} - m_\lambda) R_{\mu\nu}(k),\end{aligned}\tag{2.16}$$

where

$$R_{\mu\nu}(k) = \tilde{g}_{\mu\nu}(k) - \frac{1}{3} \tilde{\gamma}_\mu(k) \tilde{\gamma}_\nu(k),\tag{2.17}$$

with

$$\tilde{g}_{\mu\nu}(k) = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\tag{2.18}$$

and

$$\tilde{\gamma}_\mu(k) = \tilde{g}_{\mu\alpha}(k) \gamma^\alpha.\tag{2.19}$$

The tensor $R_{\mu\nu}(k)$ is transverse to k , and also satisfies the RS auxiliary conditions

$$\begin{aligned}\gamma^\mu R_{\mu\nu}(k) &= 0, \\ R_{\mu\nu}(k) \gamma^\nu &= 0.\end{aligned}\tag{2.20}$$

From the definition of $\tilde{\gamma}(k)$ in Eq. (2.19), and assuming the on-shell condition of Eq. (2.3), the following relations are readily verified,

$$\begin{aligned}(\not{k} \pm m_\lambda) \tilde{\gamma}_\mu(k) &= (\not{k} \pm m_\lambda) \left(\gamma_\mu \mp \frac{k_\mu}{m_\lambda} \right) \\ &= - \left(\gamma_\mu \pm \frac{k_\mu}{m_\lambda} \right) (\not{k} \mp m_\lambda) \\ &= -\tilde{\gamma}_\mu(k) (\not{k} \mp m_\lambda).\end{aligned}\tag{2.21}$$

These in turn imply,

$$(\not{k} \pm m_\lambda) \tilde{\gamma}_\mu(k) \tilde{\gamma}_\nu(k) = -\tilde{\gamma}_\mu(k) (\not{k} \mp m_\lambda) \tilde{\gamma}_\nu(k) = \tilde{\gamma}_\mu(k) \tilde{\gamma}_\nu(k) (\not{k} \pm m_\lambda),\tag{2.22}$$

and can be used to show the following,

$$\begin{aligned}(\not{k} \pm m_\lambda) R_{\alpha\beta}(k) &= R_{\alpha\beta}(k) (\not{k} \pm m_\lambda) \\ &= (\not{k} \pm m_\lambda) \tilde{g}_{\alpha\beta}(k) + \frac{1}{3} \left(\gamma_\alpha \pm \frac{k_\alpha}{m_\lambda} \right) (\not{k} \mp m_\lambda) \left(\gamma_\beta \pm \frac{k_\beta}{m_\lambda} \right), \\ &= (\not{k} \pm m_\lambda) \tilde{g}_{\alpha\beta}(k) - \frac{1}{3} (\not{k} \pm m_\lambda) \left(\gamma_\alpha \mp \frac{k_\alpha}{m_\lambda} \right) \left(\gamma_\beta \pm \frac{k_\beta}{m_\lambda} \right), \\ &= (\not{k} \pm m_\lambda) \tilde{g}_{\alpha\beta}(k) - \frac{1}{3} \left(\gamma_\alpha \pm \frac{k_\alpha}{m_\lambda} \right) \left(\gamma_\beta \mp \frac{k_\beta}{m_\lambda} \right) (\not{k} \pm m_\lambda),\end{aligned}\tag{2.23}$$

which are useful in the evaluation of the various traces that appear in the calculations.

C. The spin-1/2 vertex function

We parametrize the on-shell electromagnetic vertex function of the nucleons in the form

$$\Gamma_\mu^{(f)} = e [F_f(q^2)\gamma_\mu + G_f(q^2)P_\mu] . \quad (2.24)$$

F_f and G_f are given by

$$\begin{aligned} F_f &= F_f^{(1)} - 2m_f F_f^{(2)} , \\ G_f &= F_f^{(2)} , \end{aligned} \quad (2.25)$$

where the $F_f^{(i)}$ are the standard electromagnetic form factors, which are defined by writing

$$\Gamma_\mu^{(f)} = e \left[F_f^{(1)}(q^2)\gamma_\mu + iF_f^{(2)}(q^2)\sigma_{\mu\nu}q^\nu \right] , \quad (2.26)$$

and which are normalized such that

$$\begin{aligned} F_p^{(1)}(0) &= 1 , \\ F_n^{(1)}(0) &= 0 , \\ F_f^{(2)}(0) &= \frac{\kappa_f}{2m_f} , \end{aligned} \quad (2.27)$$

with $\kappa_p = 1.79$ and $\kappa_n = -1.91$.

D. The spin-3/2 vertex function

As already mentioned in the Introduction, the spin-3/2 vertex function is given by Eq. (1.4), with $g = 0$. For calculational purposes we rewrite it in the form

$$\Gamma_{\mu\alpha\beta} = fD_{\mu\alpha\beta} + \frac{if}{m_\lambda}E_{\mu\alpha\beta} , \quad (2.28)$$

where

$$D_{\mu\alpha\beta} = (q^2\gamma_\mu - q_\mu\not{q})\gamma_5 g_{\alpha\beta} , \quad (2.29)$$

and

$$E_{\mu\alpha\beta} = q_\beta e_{\alpha\mu} - q_\alpha e_{\beta\mu} , \quad (2.30)$$

with

$$e_{\alpha\mu} = \epsilon_{\alpha\mu\nu\lambda}q^\nu Q^\lambda . \quad (2.31)$$

We note the following obvious relations

$$q^\alpha e_{\alpha\mu} = Q^\alpha e_{\alpha\mu} = 0 , \quad (2.32)$$

which in turn imply

$$e_{\alpha\mu}k^\alpha = e_{\alpha\mu}k'^\alpha = 0 . \quad (2.33)$$

From these we also obtain the following useful ones

$$q_\alpha \tilde{g}^{\alpha\beta}(k)e_{\beta\mu} = q_\alpha \tilde{g}^{\alpha\beta}(k')e_{\beta\mu} = 0 , \quad (2.34)$$

and similarly with $\tilde{g}^{\alpha\beta}(k)$ replaced by $\tilde{g}^{\alpha\beta}(k')$. Moreover, by expanding the product of two epsilon symbols we obtain the formula

$$e_{\alpha\mu}e^\alpha{}_\nu = q^2Q_{\mu\nu} + Q^2q_\mu q_\nu , \quad (2.35)$$

where

$$Q_{\mu\nu} = Q_\mu Q_\nu - Q^2 g_{\mu\nu} , \quad (2.36)$$

which appears in various places in the calculations in the appendices.

III. THE ANNIHILATION PROCESS

A. The squared amplitude

The amplitude for the annihilation process $\lambda(k) + \lambda(k') \rightarrow \ell(p) + \bar{\ell}(p')$ is given by

$$iM = \frac{ie}{q^2} [\bar{v}(p', s') \gamma^\mu u(p, s)] [\bar{V}^\alpha(k', \sigma') \Gamma_{\mu\alpha\beta} U^\beta(k, \sigma)] , \quad (3.1)$$

where u, v are the spin-1/2 Dirac spinors while U, V are the spin-3/2 Rarita-Schwinger spinors, with the conventions set in Section II.

The amplitude squared, averaged over the initial spins and summed over the final spins will be expressed as the sum of three terms, corresponding to the square of the D term in Eq. (2.28), the square of the E term and the interference term, respectively. Thus,

$$\langle |M|^2 \rangle = e^2 f^2 (\mathcal{M} + \mathcal{M}' + \mathcal{M}'') , \quad (3.2)$$

where

$$\begin{aligned} \mathcal{M} &= \ell^{\mu\nu} L_{\mu\nu} , \\ \mathcal{M}' &= \left(\frac{1}{m_\lambda q^2} \right)^2 \ell^{\mu\nu} L'_{\mu\nu} , \\ \mathcal{M}'' &= \left(\frac{1}{m_\lambda q^2} \right) \ell^{\mu\nu} L''_{\mu\nu} . \end{aligned} \quad (3.3)$$

$\ell_{\mu\nu}$ is given by

$$\ell_{\mu\nu} = \frac{1}{4} \text{Tr} \gamma_\mu (\not{p} + m_\ell) \gamma_\nu (\not{p}' - m_\ell) , \quad (3.4)$$

while

$$L_{\mu\nu} = \frac{1}{4} \text{Tr} \gamma_\mu \gamma_5 \rho^{\alpha\beta}(k) \gamma_\nu \gamma_5 \bar{\rho}_{\beta\alpha}(k') , \quad (3.5)$$

$$L'_{\mu\nu} = \frac{1}{4} E_{\mu\alpha\beta} E_{\nu\sigma\tau} \text{Tr} \rho^{\beta\tau}(k) \bar{\rho}^{\sigma\alpha}(k') , \quad (3.6)$$

$$\begin{aligned} L''_{\mu\nu} &= - \left(\frac{i}{4} \right) g_{\sigma\tau} E_{\nu\alpha\beta} \text{Tr} \gamma_\mu \gamma_5 \rho^{\tau\beta}(k) \bar{\rho}^{\alpha\sigma}(k') \\ &\quad + \left(\frac{i}{4} \right) g_{\sigma\tau} E_{\mu\alpha\beta} \text{Tr} \gamma_\nu \gamma_5 \bar{\rho}^{\sigma\alpha}(k') \rho^{\beta\tau}(k) , \end{aligned} \quad (3.7)$$

where $E_{\mu\alpha\beta}$ is defined in Eq. (2.30), and the spin-3/2 projection matrices $\rho_{\mu\nu}(k), \bar{\rho}_{\mu\nu}(k')$ are given in Eq. (2.16). In the above expressions for $L_{\mu\nu}$ and $L''_{\mu\nu}$ the terms of the spin-3/2 vertex function that contain a factor of q_μ or q_ν have dropped out due to the relation (current conservation)

$$q^\mu \ell_{\mu\nu} = q^\nu \ell_{\mu\nu} = 0 . \quad (3.8)$$

$\ell_{\mu\nu}$ is of course trivially evaluated to yield

$$\ell_{\mu\nu} = p_\mu p'_\nu + p'_\mu p_\nu - \frac{1}{2} q^2 g_{\mu\nu} . \quad (3.9)$$

The evaluation of the traces that appear in the expressions for $L_{\mu\nu}, L'_{\mu\nu}, L''_{\mu\nu}$ is facilitated by using the identities and relations for the RS spinors and polarizations sums given in Section II. However, the calculations are involved and therefore we provide some details in appendices A, B and C respectively. The final results obtained in the appendices can be written in the form

$$\begin{aligned} L_{\mu\nu} &= -L Q_{\mu\nu} + O(q_\mu, q_\nu) , \\ L'_{\mu\nu} &= -(m_\lambda q^2)^2 L' Q_{\mu\nu} + O(q_\mu, q_\nu) , \\ L''_{\mu\nu} &= -(m_\lambda q^2) L'' Q_{\mu\nu} + O(q_\mu, q_\nu) , \end{aligned} \quad (3.10)$$

where $Q_{\mu\nu}$ is defined in Eq. (2.36),

$$\begin{aligned} L &= \frac{1}{3} \left[1 + \frac{2}{3} \left(\frac{Q^2}{2m_\lambda^2} - 1 \right)^2 \right], \\ L' &= \left(\frac{5}{36} \right) \left(\frac{Q^2}{m_\lambda^2} \right)^2, \\ L'' &= -\frac{Q^2(Q^2 + m_\lambda^2)}{9m_\lambda^4}, \end{aligned} \quad (3.11)$$

and $O(q_\mu, q_\nu)$ stands for terms that are proportional to q_μ and/or q_ν , which do not contribute to the squared amplitude when they are contracted with $\ell_{\mu\nu}$. Therefore, from Eqs. (3.2), (3.3) and (3.10),

$$\langle |M|^2 \rangle = e^2 f^2 (L + L' + L'') (-Q^{\mu\nu} \ell_{\mu\nu}). \quad (3.12)$$

B. The cross section

We compute the cross section and the rate in the center of mass coordinate system of the particles. The contraction formula that appears in Eq. (3.12) is readily evaluated to yield

$$(Q^2 g^{\mu\nu} - Q^\mu Q^\nu) \ell_{\mu\nu} = \frac{1}{2} (s - 4m_\lambda^2) s A \quad (3.13)$$

where

$$A = \left(1 + \frac{4m_\ell^2}{s} \right) + \left(1 - \frac{4m_\ell^2}{s} \right) \cos^2 \theta, \quad (3.14)$$

with θ being the scattering angle in the CM mass frame, i.e.,

$$\cos \theta = \hat{p} \cdot \hat{k}. \quad (3.15)$$

and $s = q^2$. Therefore, from Eqs. (3.12) and (3.13),

$$\langle |M|^2 \rangle = e^2 f^2 (L + L' + L'') \frac{1}{2} (s - 4m_\lambda^2) s A, \quad (3.16)$$

where L, L', L'' are given in Eq. (3.11), and using Eq. (2.12) can be expressed in the form

$$\begin{aligned} L &= \frac{1}{3} \left[1 + \frac{2}{3} \left(\frac{s}{2m_\lambda^2} - 1 \right)^2 \right], \\ L' &= \frac{5}{36} \left(\frac{s}{m_\lambda^2} - 4 \right)^2, \\ L'' &= -\frac{1}{9} \left(\frac{s}{m_\lambda^2} - 4 \right) \left(\frac{s}{m_\lambda^2} - 5 \right). \end{aligned} \quad (3.17)$$

From the standard formula for the cross section in the CM frame,

$$\frac{d\sigma}{d\cos\theta} = \frac{|\vec{p}|}{32\pi s |\vec{k}|} \langle |M|^2 \rangle, \quad (3.18)$$

we then obtain finally

$$\left(\frac{d\sigma}{d\cos\theta} \right)_{\lambda\lambda \rightarrow \ell\bar{\ell}} = \frac{r e^2 f^2}{32\pi} B(m_\lambda^2), \quad (3.19)$$

where we have defined

$$r = \frac{1}{2} (L + L' + L'') = \frac{1}{6} \left(\frac{s}{2m_\lambda^2} - 1 \right)^2 + \frac{1}{9}. \quad (3.20)$$

and

$$B(m^2) = \sqrt{(s - 4m^2)(s - 4m_\ell^2)} A, \quad (3.21)$$

with A given in Eq. (3.14).

C. The spin-1/2 and spin-1 cases

For reference purposes and comparison it is useful to quote the corresponding results for a spin-1/2 Majorana particle and for a self-conjugate spin-1 particle, which we denote by χ and V respectively. The corresponding electromagnetic vertex functions are given by [1, 6]

$$\Gamma_\mu^{(\chi)} = f(q^2 \gamma_\mu - q_\mu \not{q}) \gamma_5, \quad (3.22)$$

$$\Gamma_{\mu\alpha\beta}^{(V)} = if(q_\beta \epsilon_{\alpha\mu\nu\lambda} - q_\alpha \epsilon_{\beta\mu\nu\lambda}) q^\nu Q^\lambda. \quad (3.23)$$

1. spin-1/2

Using the $\Gamma_\mu^{(\chi)}$ vertex function, the amplitude for $\chi\chi \rightarrow \ell\bar{\ell}$ is

$$M = ef [\bar{u}(p) \gamma^\mu v(p')] [\bar{v}(k') \gamma_\mu \gamma_5 u(k)], \quad (3.24)$$

and taking the relevant traces

$$\langle |M|^2 \rangle = 2e^2 f^2 (-Q^{\mu\nu} \ell_{\mu\nu}), \quad (3.25)$$

which can be evaluated explicitly by using Eq. (3.13). From Eqs. (3.18) and (3.25) the annihilation cross section is then given by

$$\left(\frac{d\sigma}{d\cos\theta} \right)_{\chi\chi \rightarrow \ell\bar{\ell}} = \frac{e^2 f^2}{32\pi} B(m_\chi^2), \quad (3.26)$$

with B defined in Eq. (3.21).

2. spin-1

In analogous fashion, using the $\Gamma_{\mu\alpha\beta}^{(V)}$ vertex function the averaged squared amplitude for $VV \rightarrow \ell\bar{\ell}$ is

$$\langle |M|^2 \rangle = \frac{4}{9} \frac{e^2 f^2}{s^2} V^{\mu\nu} \ell_{\mu\nu} \quad (3.27)$$

where

$$V_{\mu\nu} = \tilde{g}^{\alpha\sigma}(k') \tilde{g}^{\beta\tau}(k) E_{\mu\alpha\beta} E_{\nu\sigma\tau}, \quad (3.28)$$

with $\tilde{g}_{\alpha\beta}(k)$ and $E_{\mu\alpha\beta}$ defined in Eqs. (2.18) and (2.30), respectively. The above expression for $V_{\mu\nu}$ coincides with the quantity defined as $K_{\mu\nu 1}$ in Eq. (B4). Borrowing the result given in Eq. (B11) and using Eq. (2.35) we then have

$$V_{\mu\nu} = -2s^2 \left(\frac{s}{4m_V^2} - 1 \right) Q_{\mu\nu} + O(q_\mu, q_\nu), \quad (3.29)$$

and therefore

$$\langle |M|^2 \rangle = \frac{8}{9} e^2 f^2 \left(\frac{s}{4m_V^2} - 1 \right) (-Q^{\mu\nu} \ell_{\mu\nu}), \quad (3.30)$$

which can be expressed explicitly by using Eq. (3.13) once more. Thus, from Eqs. (3.18) and (3.30)

$$\left(\frac{d\sigma}{d\cos\theta} \right)_{VV \rightarrow \ell\bar{\ell}} = \frac{4}{9} \left(\frac{s}{4m_V^2} - 1 \right) \frac{e^2 f^2}{32\pi} B(m_V^2). \quad (3.31)$$

D. Discussion

The angular dependence in each of the three cases is the same, and the total cross section in each one is proportional to

$$A_0 = \int d(\cos \theta) A = \frac{8}{3} + \frac{16m_\ell^2}{s}. \quad (3.32)$$

However, the energy dependence is different. For definiteness let us assume that

$$m_{\lambda,\chi,V} \gg m_\ell, \quad (3.33)$$

which is the case in most situations of practical interest. Then $A_0 \simeq \frac{8}{3}$ and defining

$$B_0(m) = \int d(\cos \theta) B(m), \quad (3.34)$$

we have

$$B_0(m) = \frac{8}{3} \times \begin{cases} 4|\vec{k}|^2 & (\text{for } |\vec{k}| \gg m) \\ 4|\vec{k}|m & (\text{for } |\vec{k}| \ll m) \end{cases} \quad (3.35)$$

where $|\vec{k}|$ is the magnitude of the momentum of one of the initial particles. The relative importance of the contributions from the two terms of the vertex function (i.e., the D and E terms in Eq. (2.28)) is given by the quantities denoted L and L' in Eq. (3.17) (L'' is the contribution from the interference term). Since their behaviour is quite different in the high or the low energy limit let us consider them separately.

1. High energy limit

In the limit $s \simeq 4|\vec{k}|^2 \gg 4m_\lambda^2$ the D and E terms give a comparable contribution,

$$\begin{aligned} L &\simeq \frac{s^2}{18m_\lambda^4}, \\ L' &\simeq \frac{5s^2}{36m_\lambda^4}, \\ L'' &\simeq -\frac{s^2}{9m_\lambda^4}. \end{aligned} \quad (3.36)$$

The total cross section in this limit is then

$$\sigma_{\lambda\lambda \rightarrow \ell\bar{\ell}} \simeq \left(\frac{s^2}{24m_\lambda^4} \right) \frac{e^2 f^2 s}{32\pi}, \quad (3.37)$$

whereas

$$\begin{aligned} \sigma_{\chi\chi \rightarrow \ell\bar{\ell}} &\simeq \frac{e^2 f^2 s}{32\pi}, \\ \sigma_{VV \rightarrow \ell\bar{\ell}} &\simeq \left(\frac{s}{9m_V} \right) \frac{e^2 f^2 s}{32\pi}. \end{aligned} \quad (3.38)$$

2. Low energy limit

In the limit $|\vec{k}|^2 \ll m_\lambda^2$,

$$\begin{aligned} L &\simeq \frac{5}{9}, \\ L' &= \frac{20|\vec{k}|^4}{9m_\lambda^4}, \\ L'' &= \frac{4|\vec{k}|^2}{9m_\lambda^2}, \end{aligned} \quad (3.39)$$

so that the E term gives a negligible contribution compared with the D term. The total cross section is then

$$\sigma_{\lambda\lambda\rightarrow\ell\bar{\ell}} \simeq \left(\frac{5}{18}\right) \frac{e^2 f^2 |\vec{k}| m_\lambda}{3\pi}, \quad (3.40)$$

which is very similar to the result for the spin-1/2 case,

$$\sigma_{\chi\chi\rightarrow\ell\bar{\ell}} \simeq \frac{e^2 f^2 |\vec{k}| m_\chi}{3\pi}. \quad (3.41)$$

Thus, for example, in the context of the calculation of the cosmological relic abundance of the λ particles, following the steps outlined in Ref. [16] in the spin-1/2 case to calculate the thermal average of the total rate $v_{rel}\sigma$, we obtain in this case

$$\langle v_{rel}\sigma_{\lambda\lambda\rightarrow\ell\bar{\ell}} \rangle = \frac{5e^2 f^2 m_\lambda T}{18\pi}, \quad (3.42)$$

which has the same temperature dependence as the in spin-1/2 case

$$\langle v_{rel}\sigma_{\chi\chi\rightarrow\ell\bar{\ell}} \rangle = \frac{e^2 f^2 m_\chi T}{\pi}. \quad (3.43)$$

IV. THE SCATTERING PROCESS

Here we consider the scattering process $\lambda(k) + f(p) \rightarrow \lambda(k') + f(p')$. In analogy with Eq. (3.2), the squared amplitude in this case can be expressed in the form

$$\langle |M|^2 \rangle = 2e^2 f^2 (\mathcal{M} + \mathcal{M}' + \mathcal{M}''), \quad (4.1)$$

where

$$\begin{aligned} \mathcal{M} &= f^{\mu\nu} \tilde{L}_{\mu\nu}, \\ \mathcal{M}' &= \left(\frac{1}{m_\lambda q^2}\right)^2 f^{\mu\nu} \tilde{L}'_{\mu\nu}, \\ \mathcal{M}'' &= \left(\frac{1}{m_\lambda q^2}\right) f^{\mu\nu} \tilde{L}''_{\mu\nu}. \end{aligned} \quad (4.2)$$

With the convention given in Eq. (2.24) for the electromagnetic vertex function of the spin-1/2 fermion, $f_{\mu\nu}$ is given by

$$f_{\mu\nu} = \frac{1}{4} \text{Tr} (F_f \gamma_\mu + G_f P_\mu) (\not{p} + m_f) (F_f \gamma_\nu + G_f P_\nu) (\not{p}' + m_f), \quad (4.3)$$

which is straightforwardly evaluated to yield

$$f_{\mu\nu} = F_f^2 \left[p_\mu p'_\nu + p'_\mu p_\nu + \frac{1}{2} q^2 g_{\mu\nu} \right] + \left[G_f^2 \left(2m_f^2 - \frac{1}{2} q^2 \right) + 2m_f F_f G_f \right] P_\mu P_\nu. \quad (4.4)$$

On the other hand, $\tilde{L}_{\mu\nu}$, $\tilde{L}'_{\mu\nu}$ and $\tilde{L}''_{\mu\nu}$ are given by expressions analogous to those for L , L' and L'' in Eq. (3.5), respectively, but with the replacement $\bar{\rho}_{\mu\nu}(k') \rightarrow \rho_{\mu\nu}(k')$, which can be expressed conveniently by the substitution rule

$$\begin{aligned} \tilde{L}_{\mu\nu} &= -L_{\mu\nu}(k' \rightarrow -k') \\ \tilde{L}'_{\mu\nu} &= -L'_{\mu\nu}(k' \rightarrow -k') \\ \tilde{L}''_{\mu\nu} &= -L''_{\mu\nu}(k' \rightarrow -k'). \end{aligned} \quad (4.5)$$

Thus, from Eq. (3.10), we have

$$\begin{aligned} \tilde{L}_{\mu\nu} &= \tilde{L} Q_{\mu\nu} + O(q_\mu, q_\nu), \\ \tilde{L}'_{\mu\nu} &= (m_\lambda q^2)^2 \tilde{L}' Q_{\mu\nu} + O(q_\mu, q_\nu), \\ \tilde{L}''_{\mu\nu} &= (m_\lambda q^2) \tilde{L}'' Q_{\mu\nu} + O(q_\mu, q_\nu), \end{aligned} \quad (4.6)$$

where

$$\begin{aligned}\tilde{L} &= \frac{1}{3} \left[1 + \frac{2}{3} \left(1 - \frac{t}{2m_\lambda^2} \right)^2 \right], \\ \tilde{L}' &= \frac{5}{36} \left(4 - \frac{t}{m_\lambda^2} \right)^2, \\ \tilde{L}'' &= -\frac{1}{9} \left(4 - \frac{t}{m_\lambda^2} \right) \left(5 - \frac{t}{m_\lambda^2} \right).\end{aligned}\tag{4.7}$$

In writing Eq. (4.7) we have used the kinematic relations given in Eq. (2.6). The expression in Eq. (4.1) for the squared amplitude then yields

$$\langle |M|^2 \rangle = 2e^2 f^2 (\tilde{L} + \tilde{L}' + \tilde{L}'') Q^{\mu\nu} f_{\mu\nu}.\tag{4.8}$$

The final ingredient to compute the cross section is the contraction $Q^{\mu\nu} f_{\mu\nu}$. Using $Q^{\mu\nu}(p - p')_\nu = 0$ we have, on one hand

$$Q^{\mu\nu} P_\mu P_\nu = 4Q^{\mu\nu} p_\mu p_\nu,\tag{4.9}$$

and on the other hand

$$Q^{\mu\nu} \left[p_\mu p'_\nu + p'_\mu p_\nu + \frac{1}{2} q^2 g_{\mu\nu} \right] = 2Q^{\mu\nu} p_\mu p_\nu + t(t - 4m_\lambda^2),\tag{4.10}$$

where $Q^{\mu\nu} p_\mu p_\nu$ is in turn is straightforwardly evaluated and it can be expressed in the form

$$Q^{\mu\nu} p_\mu p_\nu = (s - m_f^2 - m_\lambda^2)^2 + st - 4m_f^2 m_\lambda^2,\tag{4.11}$$

where we have introduced

$$s = (p + k)^2.\tag{4.12}$$

Putting these together, we then obtain

$$Q^{\mu\nu} f_{\mu\nu} = C(m_\lambda^2),\tag{4.13}$$

where

$$C(m^2) = t(t - 4m^2)F_f^2 + [2(s - m_f^2 - m^2)^2 + 2st - 8m_f^2 m^2] [F_f^2 + G_f^2(4m_f^2 - t) + 4m_f F_f G_f].\tag{4.14}$$

From the standard formula

$$\frac{d\sigma}{dt} = \frac{1}{64\pi m_f^2 |\vec{k}|^2} \langle |M|^2 \rangle,\tag{4.15}$$

the final result for the cross section is

$$\left(\frac{d\sigma}{dt} \right)_{\lambda_f \rightarrow \lambda_f} = \frac{\tilde{r} e^2 f^2}{32\pi m_f^2 |\vec{k}|^2} C(m_\lambda^2).\tag{4.16}$$

where

$$\tilde{r} = \tilde{L} + \tilde{L}' + \tilde{L}'' = \frac{1}{3} \left(1 - \frac{t}{2m_\lambda^2} \right)^2 + \frac{2}{9}.\tag{4.17}$$

Again it is useful to quote the corresponding results for the spin-1/2 Majorana (χ) and the self-conjugate spin-1 particle (V), for which the electromagnetic vertex functions are given by Eqs. (3.22) and (3.23) respectively. The results can be obtained by using as guidance the discussion in Section III C. In particular the results given in Eq. (3.25) for χ and in Eq. (3.30) for V can be reused, remembering that in the present case $\ell_{\mu\nu} \rightarrow f_{\mu\nu}$ and by properly

taking into account the changes due to the t -channel kinematics and the difference in the numerical factors that arise from the averaging (sum) over the initial (final) polarizations. Thus,

$$\left(\frac{d\sigma}{dt}\right)_{\chi f \rightarrow \chi f} = \frac{e^2 f^2}{32\pi m_f^2 |\vec{k}|^2} C(m_\chi^2), \quad (4.18)$$

is the cross section for $\chi + f \rightarrow \chi + f$, with the χ electromagnetic vertex function given by Eq. (3.22). Similarly for V , with the vertex function given by Eq. (3.23), the cross section for $V + f \rightarrow V + f$ is

$$\left(\frac{d\sigma}{dt}\right)_{Vf \rightarrow Vf} = \frac{2}{3} \left(1 - \frac{t}{4m_V^2}\right) \frac{e^2 f^2}{32\pi m_f^2 |\vec{k}|^2} C(m_V^2). \quad (4.19)$$

A. Discussion

Eqs. (4.14) and (4.16) can be used to determine the scattering cross section off a high Z nucleus N , which is a relevant quantity in the context of direct dark matter detection experiments. In this context, using the subscript N in place of f wherever the latter appears, we set

$$\begin{aligned} G_N &\rightarrow 0, \\ F_N &\rightarrow Z. \end{aligned} \quad (4.20)$$

Denoting the kinetic energy of the recoil nucleus by ϵ' , using the notation given in Eq. (2.2) for the components of the momentum vectors,

$$E' = m_N + \epsilon', \quad (4.21)$$

and t and s are given by

$$\begin{aligned} t &= -2m_N \epsilon', \\ s &= m^2 + m_N^2 + 2m_N \omega, \end{aligned} \quad (4.22)$$

in terms of the variables in the rest frame of the initial nucleus. $C(m^2)$ can then be written in the form

$$C(m^2) = 4m_N^2 Z [C_1(m^2) + C_2(m^2)], \quad (4.23)$$

where

$$\begin{aligned} C_1(m^2) &= 2(\omega^2 - m^2) + \frac{\epsilon'}{m_N} (m^2 - m_N^2 - 2m m_N), \\ C_2(m^2) &= \epsilon' [\epsilon' - (\omega - m)]. \end{aligned} \quad (4.24)$$

On the other hand, the factor \tilde{r} defined in Eq. (4.17) is given by

$$\tilde{r} = \frac{2}{9} + \frac{1}{3} \left(1 + \frac{m_N \epsilon'}{m_\lambda^2}\right)^2. \quad (4.25)$$

In this process the relative contributions from the D and E terms of the vertex function, represented by the terms \tilde{L}, \tilde{L}' in Eq. (4.17), are comparable. The differential cross section is then given as a function of ϵ' by

$$\left(\frac{d\sigma}{d\epsilon'}\right)_{\lambda N \rightarrow \lambda N} = \frac{e^2 f^2}{16\pi m_N |\vec{k}|^2} \tilde{r} C(m_\lambda^2). \quad (4.26)$$

We now consider specifically the non-relativistic limit since this is a particularly relevant situation.

Non-relativistic limit

Denoting by v the initial velocity of the λ particle, in the non-relativistic limit $C_2 = O(v^4)$, and therefore

$$C(m^2) = 2v^2 + \frac{\epsilon'}{m_N} (m^2 - m_N^2 - 2mm_N) + O(v^4). \quad (4.27)$$

The differential cross section is then given by

$$\left(\frac{d\sigma}{d\epsilon'} \right)_{\lambda N \rightarrow \lambda N} = \frac{e^2 f^2}{16\pi m_N m_\lambda^2} \tilde{r} \left[2 + \frac{\epsilon'}{m_N v^2} (m_\lambda^2 - m_N^2 - 2m_\lambda m_N) \right], \quad (4.28)$$

where $0 \leq \epsilon' \leq \epsilon'_{max}$ with

$$\epsilon'_{max} = \frac{2m_N m_\lambda^2 v^2}{(m_N + m_\lambda)^2}. \quad (4.29)$$

Using this in Eq. (4.25),

$$\tilde{r}_{max} = \frac{2}{9} + \frac{1}{3} \left(1 + \frac{2m_N^2 v^2}{(m_N + m_\lambda)^2} \right)^2, \quad (4.30)$$

and therefore we can put $\tilde{r} \simeq \frac{5}{9} + O(v^2)$ in Eq. (4.28) within the approximations made there. The corresponding cross section for the spin-1/2 χ -particle is given by Eq. (4.28), with the substitution $\tilde{r} \rightarrow 1$ and of course $m_\lambda \rightarrow m_\chi$.

V. CONCLUSIONS

In this work we have calculated the cross section for the electromagnetic pair annihilation of a spin-3/2 Majorana particle (λ) into charged leptons, and for the electromagnetic scattering of such particle off a nucleon. The calculations are based on the general structure of electromagnetic vertex function obtained in Ref. [8] for the spin-3/2 Majorana particle.

The main results of our calculations are the expressions for the annihilation and scattering cross sections, given in Eqs. (3.19) and (4.16), respectively. Comparing with the corresponding formulas for the spin-1/2 case given in Eqs. (3.26) and (4.18), reveal certain similarities. For example the angular dependence of annihilation cross section is the same in the spin-1/2 and spin-3/2 cases, and at low energies, $s \simeq 4m_\lambda^2$, both of them are proportional to the momentum $|\vec{k}|$, which is a frequently-quoted result in the context of the spin-1/2 case that is known to be due to the axial vector nature of the $\gamma_\mu \gamma_5$ coupling[5]. In Section IIID we considered in more detail this limit and gave the formulas that can be useful for example in the context of the applications similar to those considered in Refs. [16, 17] for the spin-1/2 case.

However there are important differences as well. The differences between the cross sections appear in the overall factors denoted by r and \tilde{r} in Eqs. (3.19) and (4.16) for the annihilation and scattering cross sections respectively, which are given explicitly in Eqs. (3.20) and (4.17). Thus, the energy dependence of the cross sections for the spin-3/2 and spin-1/2 cases are generally different. In addition, due to the dependence of \tilde{r} on t , the angular dependence of the scattering cross section is also different in the spin-3/2 and spin-1/2 cases.

Two other processes which could be of interest are the annihilation into photons and scattering off photons,

$$\begin{aligned} \lambda + \lambda &\rightarrow \gamma + \gamma, \\ \lambda + \gamma &\rightarrow \lambda + \gamma. \end{aligned} \quad (5.1)$$

In connection with this we wish to make the following remark. In the spin-1/2 case, the amplitudes for the analogous processes

$$\begin{aligned} \chi + \chi &\rightarrow \gamma + \gamma, \\ \chi + \gamma &\rightarrow \chi + \gamma, \end{aligned} \quad (5.2)$$

vanish at the tree-level. This follows from the fact that in both cases the amplitude contains a factor of the form

$$\epsilon^\mu (q^2 \gamma_\mu - \not{q} q_\mu) \gamma_5, \quad (5.3)$$

one such factor for each of the photons in the process in question, and such factors vanish for an on-shell photon. In contrast, in the spin-3/2 vertex function [Eq. (2.24)] the D term [Eq. (2.29)] is of similar form and also vanishes for on-shell photons, but the E term [Eq. (2.30)] does not vanish and consequently the tree-level amplitude is non-zero. This crucial distinction between the spin-1/2 and spin-3/2 cases can have interesting implications in some of the physical application contexts that we have mentioned in the Introduction. We plan to carry out the corresponding calculations for those radiative process and present them in the near future.

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Appendix A: The $L_{\mu\nu}$ trace

Using Eq. (2.16), the expression for $L_{\mu\nu}$ given in Eq. (3.5) can be rewritten in the form

$$L_{\mu\nu} = \frac{1}{4} \text{Tr} \gamma_\mu (\not{k} + m_\lambda) R^{\alpha\beta}(k) \gamma_\nu R_{\beta\alpha}(k') (\not{k}' + m_\lambda). \quad (\text{A1})$$

With the help of the relations given in Eq. (2.23), Eq. (A1) can be expressed in the form

$$L_{\mu\nu} = \sum_{i=1}^4 \frac{1}{4} \text{Tr} L_{\mu\nu}^{(i)} \quad (\text{A2})$$

where

$$\begin{aligned} L_{\mu\nu}^{(1)} &= \tilde{g}^{\alpha\beta}(k) \tilde{g}_{\beta\alpha}(k') \gamma_\mu (\not{k} + m_\lambda) \gamma_\nu (\not{k}' + m_\lambda), \\ L_{\mu\nu}^{(2)} &= -\frac{1}{3} \gamma_\mu (\not{k} + m_\lambda) \left(\gamma^\alpha - \frac{k^\alpha}{m_\lambda} \right) \left(\gamma^\beta + \frac{k_\beta}{m_\lambda} \right) \tilde{g}_{\beta\alpha}(k') \gamma_\nu (\not{k}' + m_\lambda), \\ L_{\mu\nu}^{(3)} &= -\frac{1}{3} \gamma_\mu (\not{k} + m_\lambda) \gamma_\nu \tilde{g}^{\alpha\beta}(k) \left(\gamma_\beta + \frac{k'_\beta}{m_\lambda} \right) \left(\gamma_\alpha - \frac{k'_\alpha}{m_\lambda} \right) (\not{k}' + m_\lambda), \\ L_{\mu\nu}^{(4)} &= \frac{1}{9} \left(\gamma^\alpha + \frac{k'^\alpha}{m_\lambda} \right) \gamma_\mu \left(\gamma_\alpha + \frac{k_\alpha}{m_\lambda} \right) (\not{k} - m_\lambda) \left(\gamma^\beta + \frac{k_\beta}{m_\lambda} \right) \gamma_\nu \left(\gamma_\beta + \frac{k'_\beta}{m_\lambda} \right) (\not{k}' - m_\lambda). \end{aligned} \quad (\text{A3})$$

In writing $L_{\mu\nu}^{(4)}$ we have used the relations in Eq. (2.21) together with the cyclic property of the trace.

$L_{\mu\nu}^{(1)}$ is easily shown to be given by

$$L_{\mu\nu}^{(1)} = \left[2 + \left(\frac{k \cdot k}{m_\lambda^2} \right)^2 \right] \gamma_\mu (\not{k} + m_\lambda) \gamma_\nu (\not{k}' + m_\lambda) \quad (\text{A4})$$

while using the relations

$$(\not{k} + m_\lambda) \not{k} = \not{k} (\not{k} + m_\lambda) = m_\lambda (\not{k} + m_\lambda) \quad (\text{A5})$$

and the analogous relations with $k \rightarrow k'$, $L_{\mu\nu}^{(2,3)}$ are straightforwardly reduced to

$$L_{\mu\nu}^{(3)} = L_{\mu\nu}^{(2)} = -\frac{1}{3} L_{\mu\nu}^{(1)}. \quad (\text{A6})$$

For $L_{\mu\nu}^{(4)}$, we first note the relation

$$(\not{k}' - m_\lambda) \left(\gamma^\alpha + \frac{k'^\alpha}{m_\lambda} \right) \gamma_\mu \left(\gamma_\alpha + \frac{k_\alpha}{m_\lambda} \right) (\not{k} - m_\lambda) = (\not{k}' - m_\lambda) \left[\frac{k \cdot k'}{m_\lambda^2} \gamma_\mu + \frac{2(k + k')_\mu}{m_\lambda} \right] (\not{k} - m_\lambda) \quad (\text{A7})$$

and the corresponding relation with $k \leftrightarrow k'$. These relations, together with the cyclic property of the trace then imply

$$\begin{aligned} \text{Tr } L_{\mu\nu}^{(4)} &= \frac{1}{9} \text{Tr} \left[\frac{k \cdot k'}{m_\lambda^2} \gamma_\mu + \frac{2(k+k')_\mu}{m_\lambda} \right] (\not{k} - m_\lambda) \left[\frac{k \cdot k'}{m_\lambda^2} \gamma_\nu + \frac{2(k+k')_\nu}{m_\lambda} \right] (\not{k}' - m_\lambda) \\ &= \frac{1}{9} \left(\frac{k \cdot k'}{m_\lambda^2} \right)^2 \text{Tr } \gamma_\mu (\not{k} - m_\lambda) \gamma_\nu (\not{k}' - m_\lambda) + \tilde{L}_{\mu\nu}^{(4)}, \end{aligned} \quad (\text{A8})$$

where $\tilde{L}_{\mu\nu}^{(4)}$ contains one factor of $(k+k')_\mu$ and/or $(k+k')_\nu$. Evaluating the remaining trace in Eqs. (A4) and (A8), and using Eq. (A6), we finally obtain

$$L_{\mu\nu} = \frac{2}{3} \left[1 + \frac{2}{3} \left(\frac{k \cdot k'}{m_\lambda^2} \right)^2 \right] [k_\mu k'_\nu + k'_\mu k_\nu - (k \cdot k' - m_\lambda^2) g_{\mu\nu}] + \tilde{L}_{\mu\nu}^{(4)}, \quad (\text{A9})$$

can be written in the form given in Eq. (3.10) by using Eqs. (2.9) and (2.10) to express k and k' in terms of q and Q , and using the relation

$$k_\mu k'_\nu + k'_\mu k_\nu - (k \cdot k' - m_\lambda^2) g_{\mu\nu} = -\frac{1}{2} Q_\mu Q_\nu + \frac{1}{2} Q^2 g_{\mu\nu} + \frac{1}{2} q_\mu q_\nu \quad (\text{A10})$$

Appendix B: The $L'_{\mu\nu}$ trace

Using Eq. (2.16) and the relations given in Eq. (2.21), we rewrite the expression for Eq. (3.6) in the form

$$L'_{\mu\nu} = \frac{1}{4} \text{Tr} (\not{k} + m_\lambda) K_{\mu\nu} (\not{k}' - m_\lambda) \quad (\text{B1})$$

where

$$\begin{aligned} K_{\mu\nu} &= E_{\mu\alpha\beta} E_{\nu\sigma\tau} \left[\tilde{g}^{\beta\tau}(k) - \frac{1}{3} \left(\gamma^\beta - \frac{k^\beta}{m_\lambda} \right) \left(\gamma^\tau + \frac{k^\tau}{m_\lambda} \right) \right] \\ &\times \left[\tilde{g}^{\sigma\alpha}(k') - \frac{1}{3} \left(\gamma^\sigma - \frac{k'^\sigma}{m_\lambda} \right) \left(\gamma^\alpha + \frac{k'^\alpha}{m_\lambda} \right) \right]. \end{aligned} \quad (\text{B2})$$

We express $K_{\mu\nu}$ in the form

$$K_{\mu\nu} = \sum_{i=1}^4 K_{\mu\nu i}, \quad (\text{B3})$$

where

$$\begin{aligned} K_{\mu\nu 1} &= E_{\mu\alpha\beta} E_{\nu\sigma\tau} \tilde{g}^{\beta\tau}(k) \tilde{g}^{\sigma\alpha}(k'), \\ K_{\mu\nu 2} &= E_{\mu\alpha\beta} E_{\nu\sigma\tau} \tilde{g}^{\beta\tau}(k) \left(\frac{-1}{3} \right) \left(\gamma^\sigma - \frac{k'^\sigma}{m_\lambda} \right) \left(\gamma^\alpha + \frac{k'^\alpha}{m_\lambda} \right), \\ K_{\mu\nu 3} &= E_{\mu\alpha\beta} E_{\nu\sigma\tau} \left(\frac{-1}{3} \right) \tilde{g}^{\sigma\alpha}(k') \left(\gamma^\beta - \frac{k'^\beta}{m_\lambda} \right) \left(\gamma^\tau + \frac{k'^\tau}{m_\lambda} \right), \\ K_{\mu\nu 4} &= \frac{1}{9} E_{\mu\alpha\beta} E_{\nu\sigma\tau} \left(\gamma^\beta - \frac{k^\beta}{m_\lambda} \right) \left(\gamma^\tau + \frac{k^\tau}{m_\lambda} \right) \left(\gamma^\sigma - \frac{k'^\sigma}{m_\lambda} \right) \left(\gamma^\alpha + \frac{k'^\alpha}{m_\lambda} \right). \end{aligned} \quad (\text{B4})$$

and consider each term separately.

$K_{\mu\nu 1}$ is expressed in the form

$$K_{\mu\nu 1} = K_{\mu\nu 1a} + K_{\mu\nu 1b} + K_{\mu\nu 1c} + K_{\mu\nu 1d}, \quad (\text{B5})$$

where

$$K_{\mu\nu 1a} = E_{\mu\alpha\beta} E_{\nu\sigma\tau} g^{\beta\tau} g^{\sigma\alpha},$$

$$\begin{aligned}
K_{\mu\nu 1b} &= E_{\mu\alpha\beta} E_{\nu\sigma\tau} g^{\beta\tau} \left(-\frac{k'^{\sigma} k'^{\alpha}}{m_{\lambda}^2} \right) \\
K_{\mu\nu 1c} &= E_{\mu\alpha\beta} E_{\nu\sigma\tau} g^{\sigma\alpha} \left(-\frac{k^{\beta} k^{\tau}}{m_{\lambda}^2} \right) \\
K_{\mu\nu 1d} &= E_{\mu\alpha\beta} E_{\nu\sigma\tau} \left(-\frac{k^{\beta} k^{\tau}}{m_{\lambda}^2} \right) \left(-\frac{k'^{\sigma} k'^{\alpha}}{m_{\lambda}^2} \right)
\end{aligned} \tag{B6}$$

For $K_{\mu\nu 1a}$ we obtain

$$K_{\mu\nu 1a} = 2q^2 e_{\alpha\mu} e^{\alpha}_{\nu}, \tag{B7}$$

where $e_{\alpha\mu} e^{\alpha}_{\nu}$ is given in Eq. (2.35). Similarly,

$$K_{\mu\nu 1c} = K_{\mu\nu 1b} = -\left(\frac{q^2}{2m_{\lambda}} \right)^2 e_{\alpha\mu} e^{\alpha}_{\nu} \tag{B8}$$

where we have used the relations

$$E_{\mu\alpha\beta} k^{\beta} = E_{\mu\alpha\beta} k'^{\beta} = \frac{1}{2} q^2 e_{\alpha\mu}. \tag{B9}$$

Finally,

$$K_{\mu\nu 1d} = 0, \tag{B10}$$

which follows by using Eq. (B9), together with Eq. (2.33). Summarizing,

$$K_{\mu\nu 1} = \left(\frac{Q^2 q^2}{2m_{\lambda}^2} \right) e_{\alpha\mu} e^{\alpha}_{\nu}. \tag{B11}$$

Using the trace formula

$$\text{Tr}(\not{k} + m_{\lambda})(\not{k}' - m_{\lambda}) = -2Q^2, \tag{B12}$$

we then obtain

$$\frac{1}{4} \text{Tr}(\not{k} + m_{\lambda}) K_{\mu\nu 1} (\not{k}' - m_{\lambda}) = -\frac{Q^2}{2} \left(\frac{Q^2 q^2}{2m_{\lambda}^2} \right) e_{\alpha\mu} e^{\alpha}_{\mu}, \tag{B13}$$

where $e_{\alpha\mu} e^{\alpha}_{\mu}$ is given in Eq. (2.35).

Expanding it out the expression for $K_{\mu\nu 2}$ given in Eq. (B4),

$$\begin{aligned}
K_{\mu\nu 2} &= \tilde{g}^{\beta\tau}(k) \left(\frac{-1}{3} \right) \left[(E_{\nu\sigma\tau} \gamma^{\sigma})(E_{\mu\alpha\beta} \gamma^{\alpha}) - (E_{\mu\alpha\beta} \gamma^{\alpha})(E_{\nu\sigma\tau} \frac{k'^{\sigma}}{m_{\lambda}}) \right. \\
&\quad \left. + (E_{\mu\alpha\beta} \frac{k'^{\alpha}}{m_{\lambda}})(E_{\nu\sigma\tau} \gamma^{\sigma}) - (E_{\mu\alpha\beta} \frac{k'^{\alpha}}{m_{\lambda}})(E_{\nu\sigma\tau} \frac{k'^{\sigma}}{m_{\lambda}}) \right]
\end{aligned} \tag{B14}$$

In the second and the third terms we will use the relation

$$(\not{k} + m_{\lambda}) E_{\mu\alpha\beta} \gamma^{\beta} (\not{k}' - m_{\lambda}) = (\not{k} + m_{\lambda}) [-iq_{\alpha}(q^2 \gamma_{\mu} - q_{\mu} \not{q}) \gamma_5] (\not{k}' - m_{\lambda}), \tag{B15}$$

which is proven as follows. From the definition of $E_{\mu\alpha\beta}$,

$$E_{\mu\alpha\beta} \gamma^{\beta} = \not{q} e_{\alpha\mu} - q_{\alpha} e_{\beta\mu} \gamma^{\beta}, \tag{B16}$$

and therefore

$$(\not{k} + m_{\lambda}) E_{\mu\alpha\beta} \gamma^{\beta} (\not{k}' - m_{\lambda}) = -(\not{k} + m_{\lambda}) q_{\alpha} e_{\beta\mu} \gamma^{\beta} (\not{k}' - m_{\lambda}). \tag{B17}$$

By means of the identity

$$\gamma_{\mu} \gamma_{\lambda} \gamma_{\rho} = (g_{\mu\lambda} g_{\rho\beta} - g_{\mu\rho} g_{\lambda\beta} + g_{\mu\beta} g_{\lambda\rho}) \gamma^{\beta} + i\epsilon_{\mu\lambda\rho\beta} \gamma^{\beta} \gamma_5, \tag{B18}$$

together with the relation

$$(\not{k} + m_\lambda) \gamma_\mu \not{Q} \gamma_5 (\not{k}' - m_\lambda) = (\not{k} + m_\lambda) (q^2 \gamma_\mu - q_\mu \not{Q} \gamma_5 - Q_\mu \not{Q}) \gamma_5 (\not{k}' - m_\lambda), \quad (\text{B19})$$

we obtain

$$(\not{k} + m_\lambda) e_{\beta\mu} \gamma^\beta (\not{k}' - m_\lambda) = (\not{k} + m_\lambda) i (q^2 \gamma_\mu - q_\mu \not{Q}) \gamma_5 (\not{k}' - m_\lambda), \quad (\text{B20})$$

and using this in Eq. (B17) proves the identity quoted in Eq. (B15). We note here that the analogous relations

$$\begin{aligned} (\not{k} - m_\lambda) e_{\beta\mu} \gamma^\beta (\not{k}' + m_\lambda) &= (\not{k} - m_\lambda) i (q^2 \gamma_\mu - q_\mu \not{Q}) \gamma_5 (\not{k}' + m_\lambda) \\ (\not{k}' \pm m_\lambda) e_{\beta\mu} \gamma^\beta (\not{k} \mp m_\lambda) &= (-1) (\not{k}' \pm m_\lambda) i (q^2 \gamma_\mu - q_\mu \not{Q}) \gamma_5 (\not{k} \mp m_\lambda), \end{aligned} \quad (\text{B21})$$

follow in similar fashion.

With the help of Eqs. (B15) and (B9) it follows that, when $K_{\mu\nu 2}$ is sandwiched between $(\not{k} + m_\lambda)$ and $(\not{k}' - m_\lambda)$, the second and third term vanish while the first and fourth terms give

$$(\not{k} + m_\lambda) K_{\mu\nu 2} (\not{k}' - m_\lambda) = (\not{k} + m_\lambda) \left(\frac{-1}{3} \right) \left[\tilde{g}^{\beta\tau}(k) (E_{\nu\sigma\tau} \gamma^\sigma) (E_{\mu\alpha\beta} \gamma^\alpha) - \left(\frac{q^2}{2m_\lambda} \right)^2 e_{\mu\alpha} e_\nu{}^\alpha \right] (\not{k}' - m_\lambda). \quad (\text{B22})$$

The term $e_{\mu\alpha} e_\nu{}^\alpha$ is reduced by means of Eq. (2.35), while using Eq. (2.30) the first term can be expressed in the form

$$\begin{aligned} \tilde{g}^{\beta\tau}(k) (E_{\nu\sigma\tau} \gamma^\sigma) (E_{\mu\alpha\beta} \gamma^\alpha) &= q^2 e_{\tau\nu} e_{\beta\mu} g^{\beta\tau} + (q_\beta q_\tau \tilde{g}^{\beta\tau}(k)) e_{\sigma\nu} \gamma^\sigma e_{\alpha\mu} \gamma^\alpha \\ &\quad - q_\tau e_{\beta\mu} \tilde{g}^{\beta\tau}(k) e_{\sigma\nu} \gamma^\sigma \not{Q} - q_\beta e_{\tau\nu} \tilde{g}^{\beta\tau}(k) \not{Q} e_{\alpha\mu} \gamma^\alpha. \end{aligned} \quad (\text{B23})$$

Using the relations in Eq. (2.34), it follows that the last two terms in the above equation vanish, while the remaining terms can be expressed in the form

$$\tilde{g}^{\beta\tau}(k) (E_{\nu\sigma\tau} \gamma^\sigma) (E_{\mu\alpha\beta} \gamma^\alpha) = q^2 e_{\mu\alpha} e_\nu{}^\alpha + (q_\beta q_\tau \tilde{g}^{\beta\tau}) e_{\mu\alpha} e_\nu{}^\alpha + i (q_\beta q_\tau \tilde{g}^{\beta\tau}) e_{\mu\alpha} e_{\nu\sigma} \sigma^{\alpha\sigma}. \quad (\text{B24})$$

Remembering that

$$q_\beta q_\tau \tilde{g}^{\beta\tau} = q^2 \left[1 - \frac{q^2}{4m_\lambda^2} \right] = \frac{q^2 Q^2}{4m_\lambda^2}, \quad (\text{B25})$$

the term in the square bracket in Eq. (B22) becomes

$$\left[\tilde{g}^{\beta\tau}(k) (E_{\nu\sigma\tau} \gamma^\sigma) (E_{\mu\alpha\beta} \gamma^\alpha) - \left(\frac{q^2}{2m_\lambda} \right)^2 e_{\mu\alpha} e_\nu{}^\alpha \right] = q_\beta q_\tau \tilde{g}^{\beta\tau} [2e_{\mu\alpha} e_\nu{}^\alpha + ie_{\mu\alpha} e_{\nu\beta} \sigma^{\alpha\beta}], \quad (\text{B26})$$

and therefore,

$$(\not{k} + m_\lambda) K_{\mu\nu 2} (\not{k}' - m_\lambda) = -(\not{k} + m_\lambda) \frac{q^2 Q^2}{12m_\lambda^2} [2e_{\mu\alpha} e_\nu{}^\alpha + ie_{\mu\alpha} e_{\nu\beta} \sigma^{\alpha\beta}] (\not{k}' - m_\lambda). \quad (\text{B27})$$

$K_{\mu\nu 3}$ can be reduced in similar fashion and in that case obtain

$$(\not{k} + m_\lambda) K_{\mu\nu 3} (\not{k}' - m_\lambda) = -(\not{k} + m_\lambda) \frac{q^2 Q^2}{12m_\lambda^2} [2e_{\mu\alpha} e_\nu{}^\alpha - ie_{\mu\alpha} e_{\nu\beta} \sigma^{\alpha\beta}] (\not{k}' - m_\lambda). \quad (\text{B28})$$

Therefore, from Eqs. (B27) and (B28),

$$(\not{k} + m_\lambda) (K_{\mu\nu 2} + K_{\mu\nu 3}) (\not{k}' - m_\lambda) = -\frac{q^2 Q^2}{3m_\lambda^2} e_{\mu\alpha} e_\nu{}^\alpha (\not{k} + m_\lambda) (\not{k}' - m_\lambda), \quad (\text{B29})$$

and using Eq. (B12),

$$\frac{1}{4} \text{Tr} (\not{k} + m_\lambda) (K_{\mu\nu 2} + K_{\mu\nu 3}) (\not{k}' - m_\lambda) = \frac{Q^2}{3} \frac{q^2 Q^2}{2m_\lambda^2} e_{\mu\alpha} e_\nu{}^\alpha. \quad (\text{B30})$$

For $K_{\mu\nu 4}$ we note that

$$\text{Tr}(\not{k} + m_\lambda) K_{\mu\nu 4} (\not{k}' - m_\lambda) = \frac{1}{9} \text{Tr}(\not{k} - m_\lambda) G_\nu (\not{k}' + m_\lambda) F_\mu, \quad (\text{B31})$$

where

$$\begin{aligned} F_\mu &= E_{\mu\alpha\beta} \left(\gamma^\alpha - \frac{k'^\alpha}{m_\lambda} \right) \left(\gamma^\beta + \frac{k^\beta}{m_\lambda} \right) \\ G_\nu &= E_{\nu\sigma\tau} \left(\gamma^\tau + \frac{k^\tau}{m_\lambda} \right) \left(\gamma^\sigma - \frac{k'^\sigma}{m_\lambda} \right), \end{aligned} \quad (\text{B32})$$

which using Eq. (2.30) and the relations in Eqs. (2.32) and (B9) can be written in the form

$$\begin{aligned} F_\mu &= e_{\alpha\mu} (\gamma^\alpha \not{q} - \not{q} \gamma^\alpha) + \frac{q^2}{m_\lambda} e_{\alpha\mu} \gamma^\alpha \\ G_\mu &= -e_{\alpha\mu} (\gamma^\alpha \not{q} - \not{q} \gamma^\alpha) + \frac{q^2}{m_\lambda} e_{\alpha\mu} \gamma^\alpha. \end{aligned} \quad (\text{B33})$$

With these forms it is straightforward to obtain

$$\begin{aligned} (\not{k}' + m_\lambda) F_\mu (\not{k} - m_\lambda) &= \left(\frac{q^2}{m_\lambda} - 4m_\lambda \right) (\not{k}' + m_\lambda) e_{\alpha\mu} \gamma^\alpha (\not{k} - m_\lambda) \\ (\not{k} - m_\lambda) G_\mu (\not{k}' + m_\lambda) &= \left(\frac{q^2}{m_\lambda} - 4m_\lambda \right) (\not{k} - m_\lambda) e_{\alpha\mu} \gamma^\alpha (\not{k}' + m_\lambda), \end{aligned} \quad (\text{B34})$$

and then using Eqs. (B20) and (B21) and the cyclic property of the trace operation in Eq. (B31),

$$\text{Tr}(\not{k} + m_\lambda) K_{\mu\nu 4} (\not{k}' - m_\lambda) = - \left(\frac{Q^2}{3m_\lambda} \right)^2 \text{Tr}(\not{k} - m_\lambda) i q^2 \tilde{\gamma}_\nu \gamma_5 (\not{k}' + m_\lambda) i q^2 \tilde{\gamma}_\mu \gamma_5, \quad (\text{B35})$$

where we have used Eq. (2.12). Therefore,

$$\text{Tr}(\not{k} + m_\lambda) K_{\mu\nu 4} (\not{k}' - m_\lambda) = \left(\frac{Q^2 q^2}{3m_\lambda} \right)^2 \text{Tr}(\not{k} + m_\lambda) \gamma_\nu (\not{k}' + m_\lambda) \gamma_\mu + O(q_\mu, q_\nu), \quad (\text{B36})$$

and finally taking the trace and using Eq. (A10)

$$\text{Tr}(\not{k} + m_\lambda) K_{\mu\nu 4} (\not{k}' - m_\lambda) = -2 \left(\frac{Q^2 q^2}{3m_\lambda} \right)^2 Q_{\mu\nu} + O(q_\mu, q_\nu), \quad (\text{B37})$$

where $Q_{\mu\nu}$ is defined in Eq. (2.36) and $O(q_\mu, q_\nu)$ stands for terms that are proportional to q_μ and/or q_ν .

Thus, collecting the formulas given in Eqs. (B13), (B29), and (B37), using Eq. (B12) and substituting Eq. (2.35) for $e_{\alpha\mu} e^\alpha_\nu$, we obtain

$$\begin{aligned} \frac{1}{4} \text{Tr}(\not{k} + m_\lambda) K_{\mu\nu 1} (\not{k}' - m_\lambda) &= -\frac{1}{4} \left(\frac{Q^2 q^2}{m_\lambda} \right)^2 Q_{\mu\nu} + O(q_\mu, q_\nu), \\ \frac{1}{4} \text{Tr}(\not{k} + m_\lambda) (K_{\mu\nu 2} + K_{\mu\nu 3}) (\not{k}' - m_\lambda) &= \frac{1}{6} \left(\frac{Q^2 q^2}{m_\lambda} \right)^2 Q_{\mu\nu} + O(q_\mu, q_\nu), \\ \frac{1}{4} \text{Tr}(\not{k} + m_\lambda) K_{\mu\nu 4} (\not{k}' - m_\lambda) &= -\frac{1}{18} \left(\frac{Q^2 q^2}{m_\lambda} \right)^2 Q_{\mu\nu} + O(q_\mu, q_\nu), \end{aligned} \quad (\text{B38})$$

which in turn yields

$$L'_{\mu\nu} = -\frac{5}{36} \left(\frac{Q^2 q^2}{m_\lambda} \right)^2 Q_{\mu\nu} + O(q_\mu, q_\nu). \quad (\text{B39})$$

Appendix C: The $L''_{\mu\nu}$ trace

We recall the following identity that holds for the trace of any set of Dirac matrices A_1, \dots, A_n ,

$$\text{Tr } A_1 \dots A_n = \text{Tr } A_n^c \dots A_1^c, \quad (\text{C1})$$

where

$$A_i^c \equiv C^{-1} A_i^T C, \quad (\text{C2})$$

with C being the C matrix transformation that is defined by the relation

$$C^{-1} \gamma^T C = -\gamma_\mu. \quad (\text{C3})$$

Using this, the term with the γ_ν in Eq. (3.7) can then be rewritten in the form

$$\begin{aligned} L''_{\mu\nu} = & \left(\frac{i}{4} \right) g_{\sigma\tau} E_{\nu\alpha\beta} \text{Tr } \gamma_5 \gamma_\mu (\not{k} + m_\lambda) R^{\tau\beta}(k) (\not{k}' - m_\lambda) R^{\alpha\sigma}(k') \\ & - \left(\frac{i}{4} \right) g_{\sigma\tau} E_{\mu\alpha\beta} \text{Tr } \gamma_5 \gamma_\nu (\not{k} - m_\lambda) R^{\tau\beta}(k) (\not{k}' + m_\lambda) R^{\alpha\sigma}(k'). \end{aligned} \quad (\text{C4})$$

and with help of Eq. (2.23), we can then write

$$L_{\mu\nu}^{(int)} = \left(\frac{i}{4} \right) \left(L_{\mu\nu}^{(1)} - L_{\mu\nu}^{(2)} \right), \quad (\text{C5})$$

where

$$\begin{aligned} L_{\mu\nu}^{(1)} &= E_{\nu\alpha\beta} \text{Tr } \gamma_5 \gamma_\mu (\not{k} + m_\lambda) J^{\beta\alpha}(k, k') (\not{k}' - m_\lambda) \\ L_{\mu\nu}^{(2)} &= E_{\mu\alpha\beta} \text{Tr } \gamma_5 \gamma_\nu (\not{k} - m_\lambda) J^{\beta\alpha}(-k, -k') (\not{k}' + m_\lambda), \end{aligned} \quad (\text{C6})$$

with

$$\begin{aligned} J^{\beta\alpha} = & g_{\sigma\tau} \left[\tilde{g}^{\tau\beta}(k) - \frac{1}{3} \left(\gamma^\tau - \frac{k^\tau}{m_\lambda} \right) \left(\gamma^\beta + \frac{k^\beta}{m_\lambda} \right) \right] \\ & \times \left[\tilde{g}^{\alpha\sigma}(k') - \frac{1}{3} \left(\gamma^\alpha - \frac{k'^\alpha}{m_\lambda} \right) \left(\gamma^\sigma + \frac{k'^\sigma}{m_\lambda} \right) \right]. \end{aligned} \quad (\text{C7})$$

This form makes it clear that once we evaluate $L_{\mu\nu}^{(1)}$, then we can obtain by writing

$$L_{\mu\nu}^{(2)} = L_{\nu\mu}^{(1)} \Big|_{m_\lambda \rightarrow -m_\lambda}. \quad (\text{C8})$$

We begin by evaluating the trace

$$\text{Tr } \gamma_5 \gamma^\mu (\not{k} + m_\lambda) q_\beta J^{\beta\alpha} (\not{k}' - m_\lambda) \quad (\text{C9})$$

Expanding out Eq. (C7) we have

$$\begin{aligned} q_\beta J^{\beta\alpha} &= \left[\tilde{g}_{\sigma\beta}(k) k'^\beta - \frac{1}{3} \left(\gamma_\sigma - \frac{k_\sigma}{m_\lambda} \right) \left(\not{k}' + \frac{k \cdot k'}{m_\lambda} \right) \right] \left[\tilde{g}^{\alpha\sigma}(k') - \frac{1}{3} \left(\gamma^\alpha - \frac{k'^\alpha}{m_\lambda} \right) \left(\gamma^\sigma + \frac{k'^\sigma}{m_\lambda} \right) \right] \\ &\equiv (J^\alpha)_{11} + (J^\alpha)_{12} + (J^\alpha)_{21} + (J^\alpha)_{22} \end{aligned} \quad (\text{C10})$$

where

$$\begin{aligned} (J^\alpha)_{11} &= \tilde{g}_{\sigma\beta}(k) k'^\beta \tilde{g}^{\alpha\sigma}(k') \\ (J^\alpha)_{12} &= -\frac{1}{3} \tilde{g}_{\sigma\beta}(k) k'^\beta \left(\gamma^\alpha - \frac{k'^\alpha}{m_\lambda} \right) \left(\gamma^\sigma + \frac{k'^\sigma}{m_\lambda} \right) \\ (J^\alpha)_{21} &= -\frac{1}{3} \tilde{g}^{\alpha\sigma}(k') \left(\gamma_\sigma - \frac{k_\sigma}{m_\lambda} \right) \left(\not{k}' + \frac{k \cdot k'}{m_\lambda} \right) \\ (J^\alpha)_{22} &= \frac{1}{9} \left(\gamma_\sigma - \frac{k_\sigma}{m_\lambda} \right) \left(\not{k}' + \frac{k \cdot k'}{m_\lambda} \right) \left(\gamma^\alpha - \frac{k'^\alpha}{m_\lambda} \right) \left(\gamma^\sigma + \frac{k'^\sigma}{m_\lambda} \right) \end{aligned} \quad (\text{C11})$$

Then we get

$$\text{Tr } \gamma_5 \gamma^\mu (\not{k} + m_\lambda) (J^\alpha)_{ab} (\not{k}' - m_\lambda) = C_{ab} \text{Tr } \gamma_5 \gamma_\mu (\not{k} + m_\lambda) \gamma^\alpha (\not{k}' - m_\lambda) \quad (\text{C12})$$

where

$$\begin{aligned} C_{11} &= 0 \\ C_{12} &= -\frac{1}{3} \left(\frac{k \cdot k'}{m_\lambda} \right) \left(1 - \frac{k \cdot k'}{m_\lambda^2} \right) \\ C_{21} &= \frac{1}{3} m_\lambda \left(1 - \frac{k \cdot k'}{m_\lambda^2} \right) \\ C_{22} &= \frac{1}{9} \left(\frac{k \cdot k'}{m_\lambda} \right) \left(1 - \frac{k \cdot k'}{m_\lambda^2} \right) \end{aligned} \quad (\text{C13})$$

Therefore

$$\begin{aligned} \text{Tr } \gamma_5 \gamma^\mu (\not{k} + m_\lambda) q_\beta J^{\beta\alpha} (\not{k}' - m_\lambda) &= C \text{Tr } \gamma_5 \gamma_\mu (\not{k} + m_\lambda) \gamma^\alpha (\not{k}' - m_\lambda) \\ &= C \text{Tr } \gamma_5 \gamma^\mu \not{k} \gamma^\alpha \not{k}' \\ &= 4iC \epsilon^{\mu\alpha\lambda\rho} k_\lambda k'_\rho \\ &= 2iC e_{\mu\alpha}, \end{aligned} \quad (\text{C14})$$

where

$$C = \left(\frac{k \cdot k'}{m_\lambda^2} - 1 \right) \left(\frac{2}{9} \frac{k \cdot k'}{m_\lambda} - \frac{1}{3} m_\lambda \right) = \frac{Q^2(Q^2 + m_\lambda^2)}{18m_\lambda^3}. \quad (\text{C15})$$

Similarly,

$$\begin{aligned} \text{Tr } \gamma_5 \gamma^\mu (\not{k} + m_\lambda) q_\alpha J^{\beta\alpha} (\not{k}' - m_\lambda) &= -C \text{Tr } \gamma_5 \gamma_\mu (\not{k} + m_\lambda) \gamma^\alpha (\not{k}' - m_\lambda) \\ &= -C \text{Tr } \gamma_5 \gamma^\mu \not{k} \gamma^\beta \not{k}' \\ &= -4iC \epsilon^{\mu\beta\lambda\rho} k_\lambda k'_\rho \\ &= -2iC e_{\mu\beta}. \end{aligned} \quad (\text{C16})$$

Using Eq. (C14) in Eq. (C6) we then obtain,

$$L_{\mu\nu}^{(1)} = -4iC e_{\alpha\mu} e^\alpha{}_\nu, \quad (\text{C17})$$

while Eq. (C8), yields

$$L_{\mu\nu}^{(2)} = -L_{\mu\nu}^{(1)}. \quad (\text{C18})$$

From Eq. (C5) we then obtain finally

$$L''_{\mu\nu} = \frac{Q^2(Q^2 + m_\lambda^2)}{9m_\lambda^3} e_{\alpha\mu} e^\alpha{}_\nu, \quad (\text{C19})$$

where $e_{\alpha\mu} e^\alpha{}_\nu$ is given explicitly in Eq. (2.35).

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